

SUPER-HEAVY STABILITY ISLAND WITH A SEMI-EMPIRICAL NUCLEAR MASS FORMULA

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Based on a semi-empirical nuclear mass formula, the super-heavy stability island is investigated. From the calculated shell corrections of super-heavy nuclei, the region $N = 172 - 178$, $Z = 116 - 120$ with shell corrections about -6 MeV roughly gives the position of the super-heavy stability island. The probability to synthesize nuclei with $Z = 126$ may be much smaller than that of produced super-heavy nuclei already, according to the obtained shell corrections and the proton drip line.

Keywords: super-heavy nuclei; nuclear mass formula; shell correction

1. Introduction

The synthesis of super-heavy nuclei has been studied for about half century and great achievements have been obtained.¹⁻³ However the central position of the super-heavy stability island is still uncertain. The predicted proton magic number could be $Z = 114$, 120 or 126, and the neutron number could be $N = 172$, 178 or 184, based on different mean field models and different model parameters.⁴⁻⁶ Careful calculations of the shell corrections and the binding energies for super-heavy nuclei play a key role for determination of the central position of the stability island.

In Refs.,^{6,7} we proposed a semi-empirical nuclear mass formula based on the macroscopic-microscopic method⁵ together with the Skyrme energy density functional. To extend the mass formula to super-heavy nuclei and the nuclei far from the β -stability line, we pay a special attention to study the isospin and mass dependence of the model parameters including symmetry energy coefficient and the symmetry potential. In addition, we consider the mirror nuclei constraint due to isospin symmetry in nuclear physics, which improves the precision of mass calculation significantly. In this talk,

we first briefly introduce the nuclear mass formula. Then, based on the calculations from the model, we investigate the shell corrections of super-heavy nuclei and the stability island.

2. An Improved Macroscopic-Microscopic Mass Formula

In the proposed model, the total energy of a nucleus is written as a sum of the liquid-drop energy and the Strutinsky shell correction ΔE ,

$$E(A, Z, \beta) = E_{\text{LD}}(A, Z) \prod_{k \geq 2} (1 + b_k \beta_k^2) + \Delta E(A, Z, \beta). \quad (1)$$

The liquid drop energy of a spherical nucleus $E_{\text{LD}}(A, Z)$ is described by a modified Bethe-Weizsäcker mass formula,

$$E_{\text{LD}}(A, Z) = a_v A + a_s A^{2/3} + E_C + a_{\text{sym}} I^2 A + a_{\text{pair}} A^{-1/3} \delta_{np} \quad (2)$$

with isospin asymmetry $I = (N - Z)/A$, the Coulomb term

$$E_C = a_c \frac{Z^2}{A^{1/3}} [1 - Z^{-2/3}] \quad (3)$$

and the symmetry energy coefficient,

$$a_{\text{sym}} = c_{\text{sym}} \left[1 - \frac{\kappa}{A^{1/3}} + \frac{2 - |I|}{2 + |I|A} \right]. \quad (4)$$

Here, we introduce an isospin-dependent term in the symmetry energy coefficient for a description of the Wigner term (some details will be discussed later). The a_{pair} term empirically describes the pairing effect (see Ref.⁶ for details). The terms with b_k describe the contribution of nuclear deformation to the macroscopic energy, and the mass dependence of b_k is written as,

$$b_k = \left(\frac{k}{2} \right) g_1 A^{1/3} + \left(\frac{k}{2} \right)^2 g_2 A^{-1/3}, \quad (5)$$

which greatly reduces the computation time for the calculation of deformed nuclei.

The microscopic shell correction

$$\Delta E = c_1 E_{\text{sh}} + |I| E'_{\text{sh}} \quad (6)$$

is obtained with the traditional Strutinsky procedure by setting the order $p = 6$ of the Gauss-Hermite polynomials and the smoothing parameter $\gamma = 1.2\hbar\omega_0$ with $\hbar\omega_0 = 41A^{-1/3}$ MeV. E_{sh} and E'_{sh} denote the shell energy of a nucleus and of its mirror nucleus, respectively. The additionally introduced

$|I|E'_{\text{sh}}$ term is to empirically take into account the mirror nuclei constraint ($\Delta E - \Delta E' \approx 0$, that is to say, a small value for the shell correction difference of a nucleus and its mirror nucleus due to the charge-symmetric and charge-independent nuclear force), with which the rms deviation of masses can be considerably reduced by about 10%. The single-particle levels are obtained under an axially deformed Woods-Saxon potential⁸ in which the depth V_q of the central potential ($q = p$ for protons and $q = n$ for neutrons) is written as

$$V_q = V_0 \pm V_s I \quad (7)$$

with the plus sign for neutrons and the minus sign for protons. V_s is the isospin-asymmetric part of the potential depth. We set the symmetry potential $V_s = a_{\text{sym}}$. Simultaneously, the isospin-dependent spin-orbit strength is adopted based on the Skyrme energy density functional,

$$\lambda = \lambda_0 \left(1 + \frac{N_i}{A} \right) \quad (8)$$

with $N_i = Z$ for protons and $N_i = N$ for neutrons, which strongly affects the shell structure of neutron-rich nuclei and super-heavy nuclei. In addition, we assume and set the radius $R = r_0 A^{1/3}$ and surface diffuseness a of the single particle potential of protons equal to those of neutrons for simplicity. For protons the Coulomb potential is additionally involved.

In this model, the isospin effects in both macroscopic and microscopic part of the formula are self-consistently considered, with which the number of model parameters is considerably reduced compared with the finite range droplet model. Here, we have 13 independent parameters a_v , a_s , a_c , c_{sym} , κ , a_{pair} , g_1 , g_2 , c_1 , V_0 , r_0 , a , λ_0 in the nuclear mass model. Based on the 2149 measured nuclear masses,⁹ the optimal model parameters (WS*) are obtained and listed in Table 1.

3. Details of the Model and Some Results

In this section, we will first introduce the Wigner term in the symmetry energy coefficient. Then, we will present some calculated results of nuclear masses and shell corrections. Finally, some properties of super-heavy nuclei are investigated with the proposed model.

It is known that the isospin effect plays a key role for neutron-rich nuclei and super-heavy nuclei. The nuclear binding energies, when plotted along isobaric sequences that cross the $N = Z$ locus, exhibit a slope discontinuity roughly proportional to $|I|$, which has been interpreted in terms of

Table 1. Model parameters.

parameter	WS*
a_v (MeV)	-15.6223
a_s (MeV)	18.0571
a_c (MeV)	0.7194
c_{sym} (MeV)	29.1563
κ	1.3484
a_{pair} (MeV)	-5.4423
g_1	0.00895
g_2	-0.4632
c_1	0.6297
V_0 (MeV)	-46.8784
r_0 (fm)	1.3840
a (fm)	0.7842
λ_0	26.3163

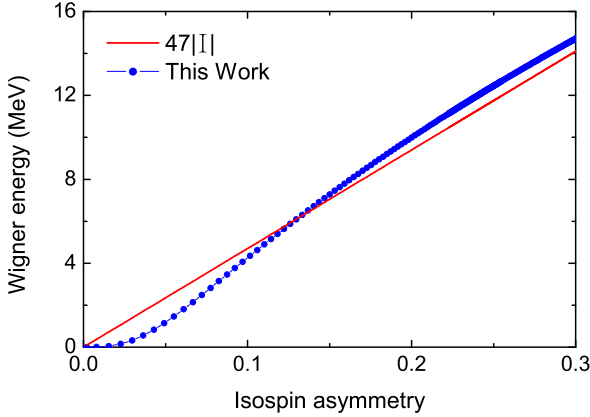


Fig. 1. (Color online) Wigner energies of nuclei along the beta-stability line. The filled circles and the straight line denote the results of this work and those of Satula et al.,¹⁰ respectively.

isospin symmetry or Wigner $SU(4)$ symmetry and is usually referred to as the Wigner effect. In this work, the Wigner effect is incorporated in the symmetry energy coefficient. The introduced I term in a_{sym} roughly leads

to a correction E_W to the binding energy of the nucleus,

$$E_W = c_{\text{sym}} I^2 A \left[\frac{2 - |I|}{2 + |I|A} \right] \approx 2c_{\text{sym}}|I| - c_{\text{sym}}|I|^2 + \dots, \quad (9)$$

which is known as the Wigner term. In Fig. 1, we show the comparison of the Wigner energies E_W of nuclei along the beta-stability line calculated with different models as a function of isospin asymmetry I . The straight line denotes the results of traditional Wigner energy about $47|I|$ MeV in Ref.¹⁰ The filled circles denote the results of this work, which is close to the results from traditional method. Compared with the case without the I term being taken into account, the rms deviation of 2149 nuclear masses can be reduced by 6%. Furthermore, when the isospin dependence of symmetry energy coefficient is taken into account, the obtained optimal c_{sym} changes from 26 to 29 MeV which is close to the calculated symmetry energy coefficient of nuclear matter at saturation density from the Skyrme energy density functional.⁶

Table 2. rms σ deviations between data AME2003⁹ and predictions of several models (in MeV). The line $\sigma(M)$ refers to all the 2149 measured masses, the line $\sigma(S_n)$ to the 1988 measured neutron separation energies S_n . The calculated masses with FRDM are taken from.⁵ The masses with HFB-14 and HFB-17 are taken from¹¹ and,¹² respectively. The results WS⁶ in our previous work in which the mirror nuclei constraint is not involved, are also listed for comparison. N_p denotes the corresponding number of parameters used in each model.

	FRDM	HFB-14	HFB-17	WS	WS*
$\sigma(M)$	0.656	0.729	0.581	0.516	0.441
$\sigma(S_n)$	0.399	0.598	0.506	0.346	0.332
N_p	31	24	24	15	15

With the obtained optimal parameters of mass formula listed in Table 1, the rms deviations of the 2149 nuclear masses is significantly reduced to 0.441 MeV and the rms deviation of the neutron separation energies of 1988 nuclei is reduced to 0.332 MeV (see Table 2). Compared with the FRDM, the rms error for the 2149 nuclear masses is considerably reduced with WS*, from 0.656 to 0.441 MeV, whilst the number of parameters in the model is reduced by a factor of two. Fig.2(a) shows the deviations between the calculated nuclear masses in this work from the experimental data. The precision of calculated masses is obviously improved in WS*, especially for

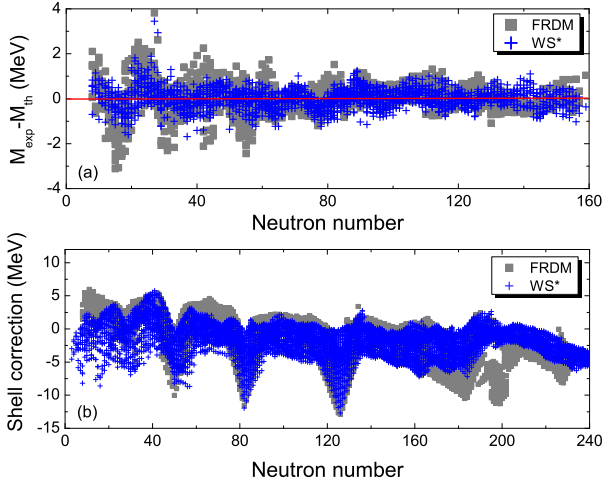


Fig. 2. (Color online) (a) Deviations between the calculated nuclear masses from the experimental data. (b) Calculated shell corrections ΔE of nuclei (crosses). The squares denote the microscopic energy of nuclei with the FRDM model (column E_{mic} of the table of Ref.⁵).

light and intermediate nuclei. In Fig.2(b), we show the calculated shell corrections ΔE of nuclei with our model and the microscopic energy (mainly including the shell correction and the deformation energy) obtained in the finite-range droplet model. For intermediate and known heavy nuclei, the results of the two approaches are comparable and both of them reproduce the known magic numbers very well. The deviations are large for light nuclei and super-heavy nuclei. The proposed model can remarkably well reproduce the shell gaps and alpha-decay energies of synthesized super-heavy nuclei (the rms deviation of the α -decay energies of 46 super-heavy nuclei is reduced to 0.263 MeV).⁷ These results give us considerable confidence to explore the super-heavy stability island.

The magic numbers 2, 8, 20, 28, 50, 82 and 126 (for neutron) are well determined from the spherical shapes of nuclei and the discontinuities of the neutron separation energy, etc. for most measured nuclei. For super-heavy nuclei, the determination of magic number however becomes a little complicated. In Fig. 3, we show the shell corrections of super-heavy nuclei. The black squares denote the nearly spherical nuclei (calculated $|\beta_2| \leq 0.01$).

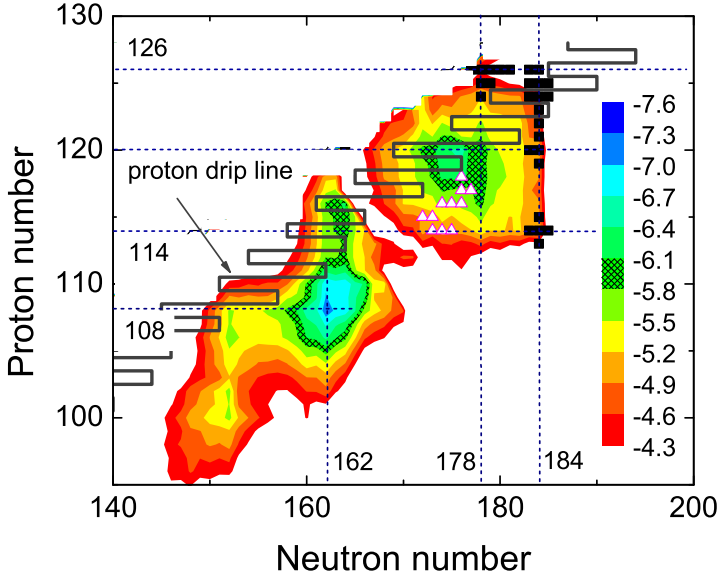


Fig. 3. (Color online) Shell correction energies of nuclei in super-heavy region from WS* calculations. The black squares denote the nearly spherical nuclei (calculated $|\beta_2| \leq 0.01$) and the triangles denote the synthesized super-heavy nuclei in the "hot" fusion reactions.^{2,3} The dark gray zigzag line denotes the calculated proton drip line. The dashed lines give the possible magic number in super-heavy region.

If to determine the magic number from the shapes in spherical of nuclei, one can see that neutron number $N = 184$ is an obvious magic number. However, the largest shell corrections of nuclei in super-heavy region locate around $N = 162$ and 178 . The results of WS* and FRDM indicate that ^{270}Hs ($N = 162$ and $Z = 108$) is a deformed doubly-magic nucleus from the large shell correction in absolute value. The shades in Fig.3 show the region ($N = 172 - 178$, $Z = 116 - 120$) of nuclei with shell corrections of about -6 MeV, which roughly gives the boundaries of the super-heavy island based on the calculated shell correction of nuclei with WS*. In addition, one can see from the calculated proton drip line that nuclei with $Z = 126$ and $N \leq 184$ locate beyond the proton drip line and the corresponding shell corrections (in absolute value) are much smaller than those

of known super-heavy nuclei, which indicates that the probability to synthesize nuclei with $Z = 126$ would be much smaller than that of produced super-heavy nuclei already.

4. Summary

Based on our proposed nuclear mass formula, we investigated the super-heavy stability island. According to the calculated masses, the rms deviation with respect to 2149 measured nuclear masses is reduced to 0.441 MeV and the rms deviation of the neutron separation energies of 1988 nuclei falls to 0.332 MeV, which give us considerable confidence to study the properties of super-heavy nuclei. From the calculated shell corrections of super-heavy nuclei, the region $N = 172 - 178$, $Z = 116 - 120$ with shell corrections about -6 MeV roughly gives the position of the super-heavy stability island. The probability to synthesize nuclei with $Z = 126$ may be much smaller than that of produced super-heavy nuclei already, since nuclei with $Z = 126$ and $N \leq 184$ locate beyond the proton drip line and the corresponding shell corrections (in absolute value) are much smaller than those of known super-heavy nuclei. To improve the reliability of the predication on super-heavy nuclei, one needs to further improve the nuclear mass model. Our preliminary results show that the rms deviation with respect to 2149 measured nuclear masses can be reduced to about 340 keV after some residual corrections are considered in the model, and the obtained position of the super-heavy stability island is generally unchanged.

5. Acknowledgments

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